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SOLUCIONES DEL EXAMEN FINAL DE SEPTIEMBRE DE ALGEBRA LINEAL

Solución 1.-

$$\det(A-\omega I)=\begin{vmatrix}-\omega & 1 & 1 & \dots & 1 & 1\\ 1 & -\omega & 1 & \dots & 1 & 1\\ 1 & 1 & -\omega & \dots & 1 & 1\\ & & & \dots & & \\ 1 & 1 & 1 & \dots & -\omega & 1\\ 1 & 1 & 1 & \dots & 1 & -\omega\end{vmatrix}=\begin{vmatrix}n-1-\omega & 1 & 1 & \dots & 1 & 1\\ n-1-\omega & -\omega & 1 & \dots & 1 & 1\\ n-1-\omega & 1 & -\omega & \dots & 1 & 1\\ & & & \dots & & \\ n-1-\omega & 1 & 1 & \dots & -\omega & 1\\ n-1-\omega & 1 & 1 & \dots & 1 & -\omega\end{vmatrix}=$$

$$=\begin{vmatrix}n-1-\omega & 1 & 1 & \dots & 1 & 1\\ 0 & -(\omega+1) & 0 & \dots & 0 & 0\\ 0 & 0 & -(\omega+1) & \dots & 0 & 0\\ & & & \dots & & \\ 0 & 0 & 0 & \dots & -(\omega+1) & 0\\ 0 & 0 & 0 & \dots & 0 & -(\omega+1)\end{vmatrix}=(-1)^{n-1}.(\omega+1)^{n-1}.(n-1-\omega).$$

$$P(\lambda)=\det(A-\lambda I)=(-1)^{n-1}.(\lambda+1)^{n-1}.(n-1-\lambda)\begin{cases}\lambda_1=-1, & \alpha_1=n-1,\\ \lambda_2=n-1, & \alpha_2=1.\end{cases}$$

$$\lambda_1=-1,\quad \begin{cases}S_1= & \{(\rho_1,\rho_2,\dots,\rho_{n-1},-\rho_1-\rho_2-\dots-\rho_{n-1})\};\\ B_{S_1}= & \{(1,0,\dots,0,-1),(0,1,\dots,0,-1),\dots,(0,0,\dots,1,-1)\};\\ \delta_1= & \dim S_1=n-1.\end{cases}$$

$$\lambda_2=n-1,\quad \begin{cases}S_2= & \{(\rho,\rho,\dots,\rho,\rho)\};\\ B_{S_2}= & \{(1,1,\dots,1,1)\};\\ \delta_2= & \dim S_2=1.\end{cases}$$

A es diagonalizable ya que

$$\sum_{i=1}^{2}\delta_i(\text{ multiplicidad geométrica })=\sum_{i=1}^{2}\alpha_i(\text{ multiplicidad algebraica })=(n-1)+1=n.$$

$$\begin{bmatrix}-1 & 0 & 0 & \dots & 0 & 0\\ 0 & -1 & 0 & \dots & 0 & 0\\ 0 & 0 & -1 & \dots & 0 & 0\\ & & & \dots & & \\ 0 & 0 & 0 & \dots & -1 & 0\\ 0 & 0 & 0 & \dots & 0 & n-1\end{bmatrix}$$